Problem 50

A student is trying to remember some formulas from geometry. In what follows, assume A is area, V is volume, and all other variables are lengths. Determine which formulas are dimensionally consistent. (a) $V = \pi r^2 h$; (b) $A = 2\pi r^2 + 2\pi rh$; (c) $V = 0.5bh$; (d) $V = \pi d^2$; (e) $V = \pi d^3/6$.

Solution

Part (a)

Check the units of both sides.

$$\begin{aligned}[V] &\stackrel{?}{=} [\pi r^2 h] \\ &\stackrel{?}{=} [\pi][r^2][h] \\ &\stackrel{?}{=} 1 \cdot \mathrm{L}^2 \cdot \mathrm{L} \\ &= \mathrm{L}^3\end{aligned}$$

Volume has dimensions of length cubed, so this equation is dimensionally consistent.

Part (b)

Check the units of both sides.

$$\begin{aligned}[A] &\stackrel{?}{=} [2\pi r^2 + 2\pi rh] \\ &\stackrel{?}{=} [2\pi r^2] + [2\pi rh] \\ &\stackrel{?}{=} [2\pi][r^2] + [2\pi][r][h] \\ &\stackrel{?}{=} 1 \cdot \mathrm{L}^2 + 1 \cdot \mathrm{L} \cdot \mathrm{L} \\ &\stackrel{?}{=} \mathrm{L}^2 + \mathrm{L}^2 \\ &= 2\mathrm{L}^2\end{aligned}$$

Area has dimensions of length squared, so this equation is dimensionally consistent. The coefficient 2 is insignificant.

Part (c)

Check the units of both sides.

$$\begin{aligned}
[V] &\stackrel{?}{=} [0.5bh] \\
&\stackrel{?}{=} [0.5][b][h] \\
&\stackrel{?}{=} 1 \cdot \mathrm{L} \cdot \mathrm{L} \\
&\neq \mathrm{L}^2
\end{aligned}$$

Volume has dimensions of length cubed, so this equation is not dimensionally consistent.

Part (d)

Check the units of both sides.

$$\begin{aligned}
[V] &\stackrel{?}{=} [\pi d^2] \\
&\stackrel{?}{=} [\pi][d^2] \\
&\stackrel{?}{=} 1 \cdot \mathrm{L}^2 \\
&\neq \mathrm{L}^2
\end{aligned}$$

Volume has dimensions of length cubed, so this equation is not dimensionally consistent.

Part (e)

Check the units of both sides.

$$\begin{aligned}
[V] &\stackrel{?}{=} \left[\frac{\pi}{6} d^3\right] \\
&\stackrel{?}{=} \left[\frac{\pi}{6}\right][d^3] \\
&\stackrel{?}{=} 1 \cdot \mathrm{L}^3 \\
&= \mathrm{L}^3
\end{aligned}$$

Volume has dimensions of length cubed, so this equation is dimensionally consistent.